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Equations of motion in linearised gravity: charged rotating sources

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Abstract. We consider the linearised gravitational and electromagnetic fields of an axially symmetric charged source which is rotating about its symmetry axis and moving with arbitrary acceleration along its symmetry axis when viewed in the flat background space–time. We establish that if the linearised field of the body and the linearised twist of the degenerate principal null direction of the Weyl tensor are 'wire' singularity-free, then the body must either move with zero acceleration or perform runaway motion from an unaccelerated state in the infinite past, and its mass and charge are constant at lowest order. Also, its rotation is either uniform or singular in the infinite past.

1. Introduction

In two recent papers, Hogan and O'Brien (1979) and O'Brien (1979) (hereinafter referred to as I and II, respectively), the Robinson and Robinson (1969) linearised fields of rotating sources, moving with arbitrary acceleration in a background Minkowskian space–time, were studied. The sources considered were of small mass, axially symmetric, rotating slowly about the symmetry axis and moving along the symmetry axis when viewed in the background space–time. The approach used was one developed by Hogan and Imaeda (1979a, b, c) to study the motion of point sources of Robinson–Trautman (1962) fields. In I and II it was established that if the linearised field of the body and the linearised twist of the degenerate principal null direction of the Riemann tensor are 'directional' singularity-free, then the body must move with uniform acceleration and, to first order, its mass must be constant and its rotation uniform.

Robinson *et al* (1969) extended the Robinson and Robinson (1969) work to a situation in which there is a source-free electromagnetic field. They presented a family of algebraically special vacuum Einstein–Maxwell line elements with a twisting, diverging, degenerate principal null direction which is also a principal null direction of the electromagnetic field. Their solutions include as special cases the Robinson–Trautman (1962) family of line elements and the Newman *et al* (1965) line element for a charged, rotating, non-accelerating, axially symmetric body. In analogy with the interpretation given in I and II of the Robinson–Robinson (1969) fields, we think of the Robinson *et al* (1969) fields as including the fields of arbitrarily moving, rotating, charged bodies. The purpose of the present paper is to examine a member of the Robinson *et al* (1969) family of line elements with this interpretation in mind.

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In § 2 we introduce the exact Robinson *et al* (1969) solution of the Einstein–Maxwell vacuum field equations. In § 3 the background Minkowskian space–time is briefly discussed and the linearisation procedure explained. In § 4 we solve the linearised field equations, determining functions of integration by the requirement that the linearised field of the body be 'wire' singularity-free. In § 5 we present the final form of the line element and discuss our results.

2. The Robinson *et al* line element

We consider a space–time which admits a null vector field k^i $(i = 1, 2, 3, 4)$ tangent to a shear-free diverging congruence of affinely parametrised null geodesic curves. Robinson and Robinson (1969) have shown that coordinates $x^i = (\zeta, \bar{\zeta}, \sigma, \rho)$ may be chosen such that the line element built around the congruence has the form

$$ds^2 = 2P\bar{P}\,d\zeta\,d\bar{\zeta} + 2\,d\Sigma(d\rho + Z\,d\zeta + \bar{Z}\,d\bar{\zeta} + S\,d\Sigma) \tag{2.1a}$$

$$d\Sigma = -(d\sigma + i\,\partial B\,d\zeta - i\,\bar{\partial}B\,d\bar{\zeta}) = k_i\,dx^i \tag{2.1b}$$

where $B = B(\zeta, \bar{\zeta}, \sigma)$, a bar denotes complex conjugation and for any function $f(\zeta, \bar{\zeta}, \sigma)$, $\partial f = \partial f/\partial \zeta$ and $\bar{\partial} f = \partial f/\partial \bar{\zeta}$. Assuming that k^i is a principal null direction of the electromagnetic field it follows that in these coordinates F_{ab} has the form

$$F_{ab} = \frac{V}{P} k_{[a}m_{b]} + \frac{\bar{V}}{\bar{P}} k_{[a}\bar{m}_{b]} + (Q + \bar{Q})k_{[a}l_{b]} + (Q - \bar{Q})m_{[a}\bar{m}_{b]} \tag{2.2}$$

where V and Q are arbitrary functions of the four coordinates and

$$m_i\,dx^i = P\,d\zeta \qquad l_i\,dx^i = d\rho + Z\,d\zeta + \bar{Z}\,d\bar{\zeta} + S\,d\Sigma. \tag{2.3}$$

Robinson *et al* (1969) have shown that the Einstein–Maxwell vacuum field equations and Maxwell's equations are satisfied provided

$$P = e^u(\rho + i\Omega) \qquad u = u(\zeta, \bar{\zeta}, \sigma) \tag{2.4a}$$

$$Z = \rho\Lambda - i(D + \Lambda)\Omega \tag{2.4b}$$

$$\Omega = -\tfrac{1}{2}e^{-2u}\{D\bar{\partial} + \bar{D}\,\partial\}B \tag{2.4c}$$

$$\Lambda = -i\,\partial\dot{B} \tag{2.4d}$$

$$Q = \frac{q}{(\rho + i\Omega)^2} \tag{2.4e}$$

$$V = v - \frac{Dq}{(\rho + i\Omega)} + \frac{2q}{(\rho + i\Omega)^2}\{iD\Omega - \rho\Lambda\} \tag{2.4f}$$

$$(\bar{D} + 2\bar{\Lambda})q = 0 \tag{2.4g}$$

$$(\bar{D} + \bar{\Lambda})v - (e^{2u}q)^{\cdot} = 0 \tag{2.4h}$$

$$S = -\rho\dot{u} - \tfrac{1}{2}K + (\rho m + \Omega M - \tfrac{1}{2}q\bar{q})/(\rho^2 + \Omega^2)^{-1} \tag{2.4i}$$

$$K = e^{-2u}(\bar{D}L + D\bar{L}) \tag{2.4j}$$

$$L = \Lambda - Du \tag{2.4k}$$

$$M = K\Omega + \tfrac{1}{2}e^{-2u}\{(\bar{D} + \bar{\Lambda})(D + \Lambda) + (D + \Lambda)(\bar{D} + \bar{\Lambda})\}\Omega \tag{2.4l}$$

$$(D+3\Lambda)(m-\mathrm{i}M)=\bar{q}v \tag{2.4m}$$

$$\mathrm{e}^{-4u}I-\mathrm{e}^{-3u}\{\mathrm{e}^{3u}(m+\mathrm{i}M)\}^{\cdot}=\tfrac{1}{2}\mathrm{e}^{-2u}v\bar{v} \tag{2.4n}$$

$$I=(\bar{D}^2+2\bar{L}\bar{D})(DL+L^2) \tag{2.4o}$$

where q, v and m are functions of ζ, $\bar{\zeta}$, σ only and for any function $f(\zeta,\bar{\zeta},\sigma)$ we use the notation $Df=\partial f-\mathrm{i}\,\partial B\dot{f}$ and $\bar{D}f=\bar{\partial}f+\mathrm{i}\,\bar{\partial}B\dot{f}$ with a dot denoting partial differentiation with respect to σ.

3. Linearisation

We proceed as in I and II and expand the exact line element (2.1) about a background Minkowskian space–time. We begin with the Minkowskian line element which is given explicitly by

$$\mathrm{d}s_0^2=2\rho^2\exp(2\underset{0}{u})\,\mathrm{d}\zeta\,\mathrm{d}\bar{\zeta}-2\,\mathrm{d}\rho\,\mathrm{d}\sigma-(1+2\underset{0}{\dot{u}}\rho)\,\mathrm{d}\sigma^2 \tag{3.1}$$

where

$$\begin{aligned}\exp(-\underset{0}{u})&=\lambda^4(1+\tfrac{1}{2}\zeta\bar{\zeta})-\lambda^3(1-\tfrac{1}{2}\zeta\bar{\zeta})-(\lambda^1-\mathrm{i}\lambda^2)\frac{\zeta}{\surd 2}-(\lambda^1+\mathrm{i}\lambda^2)\frac{\bar{\zeta}}{\surd 2}-1\\&=(\lambda^1)^2+(\lambda^2)^2+(\lambda^3)^2-(\lambda^4)^2\\ \underset{0}{\dot{u}}&=\mu^ik_i.\end{aligned} \tag{3.2}$$

From (3.1) we deduce that $\rho=0$ is a time-like world line along which σ is the proper time. The λ^i and μ^i are functions of σ only and are its 4-velocity and 4-acceleration, respectively. For a detailed derivation of (3.1) and (3.2) the reader is referred to Hogan and Imaeda (1979a).

We assume that the function m which appears in the metric (2.1) through (2.4i) is small of first order, writing $m=O_1$. We take m to be the mass $\tilde{m}$ of the source plus a term of electromagnetic origin the form of which is to be determined by (2.4). The function $q\bar{q}$ in (2.4i) is interpreted as the square of the charge of the source. An expansion for u and B of the form

$$u=\underset{0}{u}+\underset{1}{u} \tag{3.3a}$$

$$B=\underset{1}{B}+\underset{2}{B} \tag{3.3b}$$

is assumed, where a subscript n ($n=0,\frac{1}{2},1,\ldots$) under any quantity means that it is small of nth order. Substituting these into (2.4c), (2.4d), (2.4j) and (2.4k) and using (3.2) we have

$$K=1+\underset{1}{K} \tag{3.4a}$$

$$\Omega=-\tfrac{1}{2}\Delta\underset{1}{B}+\underset{2}{\Omega} \tag{3.4b}$$

where $\Delta=2\exp(-2\underset{0}{u})\,\partial^2/\partial\zeta\,\partial\bar{\zeta}$. It is assumed that the subscript zero refers to the

Minkowskian values of the various quantities. Substitution of the above expression into (2.41) now yields

$$M=\tfrac{1}{2}\Delta(\underset{1}{\Omega}-\underset{1}{B})+O_2=O_1. \tag{3.5}$$

We consider now the orders of magnitude of the electromagnetic quantities q and v. From (2.4m) it follows that $\bar{q}v=O_1$ (since $m=O_1$ and using (3.5)). Also the zeroth-order term in the left-hand side of (2.4n) vanishes so that $v\bar{v}=O_1$. It follows therefore that $v=O_{1/2}$ and $q=O_{1/2}$ and an expansion for these of the form

$$q=\underset{1/2}{q}+\underset{3/2}{q} \tag{3.6a}$$

$$v=\underset{1/2}{v}+\underset{3/2}{v} \tag{3.6b}$$

is assumed.

4. The charged source

We shall assume that the source is axially symmetric with the X^3 axis as symmetry axis and that it moves along this axis, i.e. $\lambda^1=\lambda^2=0$. Then if $\alpha=\alpha(\sigma)$ is defined by $e^\alpha=\lambda^3+\lambda^4$, (3.2) becomes

$$\exp(-\underset{0}{u})=e^\alpha(\tfrac{1}{2}\zeta\bar{\zeta}+e^{-2\alpha}) \tag{4.1a}$$

$$\underset{0}{\dot{u}}=-\dot{\alpha}\frac{(\frac{1}{2}\zeta\bar{\zeta}-e^{-2\alpha})}{(\frac{1}{2}\zeta\bar{\zeta}+e^{-2\alpha})} \tag{4.1b}$$

and $\dot{\alpha}=(\mu^i\mu_i)^{1/2}$ is the magnitude of the 4-acceleration of the source. We guarantee the axial symmetry of the source by requiring that the vector $i(\zeta\partial-\bar{\zeta}\bar{\partial})$ should satisfy Killing's equations. Clearly a sufficient condition for this is that the functions m, u, B and $q\bar{q}$, which appear in (2.1), depend on ζ and $\bar{\zeta}$ only in the combination $\bar{\zeta}\zeta$. We assume also that both q and $\bar{q}$ are axially symmetric.

Following the approach used in considering an uncharged source in I and II, we choose $\underset{1}{\Omega}=\underset{1}{B}$, with a view to the recovery of the linearised Newman *et al* (1965) solution as a special case. We require that Ω be 'wire' singularity-free thereby ensuring that the linearised twist of the vector field (2.1b) is also 'wire' singularity-free. This leads us to choose as solution to (3.4b)

$$\underset{1}{\Omega}=c(\sigma)\xi \tag{4.2}$$

where $c=O_1$ and we have introduced the new variable

$$\xi=\frac{\frac{1}{2}\zeta\bar{\zeta}-e^{-2\alpha}}{\frac{1}{2}\zeta\bar{\zeta}+e^{-2\alpha}}. \tag{4.3}$$

In order that (2.4g), (2.4h) and (2.4m) be satisfied at lowest order we must take

$$q=\epsilon(\sigma)=\bar{q} \tag{4.4a}$$

$$v=\frac{1}{\zeta}\{\dot{\epsilon}\xi+\dot{\alpha}\epsilon(1-\xi^2)+z(\sigma)\} \tag{4.4b}$$

$$m = \tilde{m}(\sigma) + 2\epsilon^2\dot{\alpha}\xi - \epsilon\dot{\epsilon}\log(1-\xi^2) + \epsilon z(\sigma)\log\left(\frac{1+\xi}{1-\xi}\right) + O_2 \tag{4.4c}$$

where $z = O_{1/2}$ is a function of integration and $\tilde{m}$ and ϵ are the mass and charge of the source, respectively, at lowest order. In the special case of $\alpha = 0$ and m, c and ϵ each having constant values (and $\underset{1}{u} = 0 = \underset{2}{B}$), the line element (2.1) becomes the Newman *et al* (1965) solution. In general, we interpret (2.1) (with the above restrictions) as the line element associated with a slowly rotating, arbitrarily accelerating, axially symmetric source, rotating non-uniformly about and moving along its symmetry axis when viewed in the flat background space–time.

In the Newman and Penrose (1966) notation, the tetrad components of the Maxwell tensor and the linearised Weyl tensor are given respectively by

$$\Phi_0 = 0 \tag{4.5a}$$

$$\Phi_1 = \frac{-\bar{q}}{2(\rho - \mathrm{i}\Omega)^2} \tag{4.5b}$$

$$\Phi_2 = \frac{\mathrm{e}^{-u}}{(\rho+\mathrm{i}\Omega)}\left(v - \frac{Dq}{(\rho+\mathrm{i}\Omega)} + \frac{2q(\mathrm{i}D\Omega - \rho\Lambda)}{(\rho+\mathrm{i}\Omega)^2}\right) \tag{4.5c}$$

and

$$\Psi_0 = \Psi_1 = 0 \tag{4.6a}$$

$$\Psi_2 = \frac{1}{(\rho+\mathrm{i}\Omega)^3}\left(-m + \frac{q\bar{q}}{(\rho-\mathrm{i}\Omega)}\right) + O_2 \tag{4.6b}$$

$$\Psi_3 = \frac{\bar{\zeta}\exp(\underset{0}{u})}{2(\rho+\mathrm{i}\Omega)^2}\left(\frac{\partial \underset{1}{K}}{\partial\xi} - \frac{3\rho}{(\rho^2+\Omega^2)}\frac{\partial m}{\partial\xi}\right) + O_2 \tag{4.6c}$$

$$\Psi_4 = \frac{\bar{\zeta}^2\exp(2\underset{0}{u})}{(\rho+\mathrm{i}\Omega)}\frac{\partial}{\partial\xi}\left(\frac{\partial\underset{1}{\dot{u}}}{\partial\xi} + 2\dot{\alpha}\underset{1}{u}\right) + \frac{\bar{\zeta}^2\exp(2\underset{0}{u})}{2(\rho+\mathrm{i}\Omega)^2}\left(\frac{\partial^2\underset{1}{K}}{\partial\xi^2} - \frac{2\rho}{(\rho^2+\Omega^2)}\frac{\partial^2 m}{\partial\xi^2}\right) + O_2. \tag{4.6d}$$

All of these tetrad components which are non-zero are singular on $\rho = 0$, $\Omega = 0$—the analogue of the 'Kerr circle'.

Using (2,4j), (2.4n), (2.4o), (4.1) and (4.2) we have

$$\underset{1}{K} = -\Delta\underset{1}{u} - 2\underset{1}{u} + O_2 \tag{4.7a}$$

$$\tfrac{1}{4}\Delta\underset{1}{K} = \dot{m} + 3m\underset{0}{\dot{u}} + \tfrac{1}{2}\exp(-2\underset{0}{u})v\bar{v} + O_2. \tag{4.7b}$$

Requiring that (4.5) and (4.6) be free of 'wire' singularities, we solve (4.7b) for $\underset{1}{K}$ with $\underset{0}{u}$, v and m given by (4.1b), (4.4b) and (4.4c) then (4.7a) for $\underset{1}{u}$. Thus the line element (2.1) is fully determined with an O_2 error.

We find that equations (4.7) are satisfied and (4.5) and (4.6) are 'wire' singularity-free at lowest order provided the function z which appears in (4.4b) and (4.4c) vanishes and $\tilde{m}$, ϵ and $3\dot{\alpha}\tilde{m} - 2\epsilon^2\ddot{\alpha}$ are constants. When $\epsilon \neq 0$, $\dot{\alpha}$, the magnitude of the 4-acceleration of the source, must be of the form

$$\dot{\alpha} = a + b\exp(3\tilde{m}\sigma/2\epsilon^2) \tag{4.8}$$

with a and b constant. When $\epsilon = 0$, the acceleration must be uniform. This case has been treated in detail in I and II. Also, we have that

$$\underset{1}{K} = 6\tilde{m}a\xi + 6\epsilon^2\dot{\alpha}^2\xi^2 + R(\sigma) + O_2 \tag{4.9a}$$

$$\underset{1}{u} = \tilde{m}a\xi \log(1-\xi^2) - \tfrac{3}{2}\epsilon^2\dot{\alpha}^2(1-\xi^2) - \tfrac{1}{2}R(\sigma)$$

$$+ \tau(\sigma)\xi + \delta\left[-1 + \tfrac{1}{2}\xi \log\left(\frac{1+\xi}{1-\xi}\right)\right] + O_2 \tag{4.9b}$$

with δ constant.

We now solve (2.4g) and (2.4h) for $\underset{3/2}{q}$ and $\underset{3/2}{v}$ respectively, determining the functions of integration by requiring that the tetrad components (4.5) be 'wire' singularity-free. This also establishes that the constants δ in (4.9b) and a in (4.8) vanish so that the acceleration is given by

$$\dot{\alpha} = b \exp(3\tilde{m}\sigma/2\epsilon^2). \tag{4.10}$$

Thus the requirement that the linearised field of the body be 'wire' singularity-free implies that either the body performs runaway motion from a state of zero acceleration in the infinite past or its acceleration vanishes. Because of the singularity in α in the infinite future and the corresponding breakdown of the perturbation, we must confine our study to the future-bounded time interval $-\infty < \sigma < \sigma_0$ for some small $\sigma_0 > 0$.

Having calculated $\underset{3/2}{q}$ and $\underset{3/2}{v}$ we can now evaluate the lowest-order non-vanishing terms in M from (2.4m). Then substituting in (2.4l) and integrating yields an expression for $\underset{2}{\Omega}$ which is singular on $\xi = \pm 1$ unless $c + (3\tilde{m}/2\epsilon^2)\dot{c} = 0$. At the outset we specified that Ω and thus the linearised twist of the vector field (2.1b) be free of 'wire' singularities, and therefore the rotation parameter c must be of the form

$$c = c' + c'' \exp(-3\tilde{m}\sigma/2\epsilon^2) \tag{4.11}$$

where c' and c'' are constants. If $c'' \neq 0$, the second term in (4.11) is singular in the infinite past and vanishes in the infinite future. It follows from (4.10) and (4.11) that, in general, the source has infinite rotational and zero translational energy in the limit $\sigma \to -\infty$ and finite rotational and infinite translational energy in the limit $\sigma \to \infty$. If $c'' = 0$, the rotation is always uniform and the approximation is valid throughout the future-bounded time interval $-\infty < \sigma < \sigma_0$ for some small $\sigma_0 > 0$. In this case, since $\dot{\alpha} \to 0$ as $\sigma \to -\infty$, we recover the linearised Newman *et al* (1965) solution in the infinite past. When $c'' \neq 0$ the approximation breaks down in the limits $\sigma \to -\infty$ and $\sigma \to \infty$ and we must confine our study to a small interval centred about the origin, i.e. $-\sigma_1 < \sigma < \sigma_1$ for some small $\sigma_1 > 0$.

The lowest-order non-zero tetrad components of the linearised Weyl and Maxwell tensors are given respectively by

$$\Psi_2 = \frac{1}{(\rho + \mathrm{i}c\xi)^3}\left(-\tilde{m} - 2\epsilon^2\dot{\alpha}\xi + \frac{\epsilon^2}{\rho - \mathrm{i}c\xi}\right) + O_2$$

$$\Psi_3 = \frac{\bar{\zeta}\exp(\underset{0}{u})}{(\rho + \mathrm{i}c\xi)^2}\left(6\epsilon^2\dot{\alpha}^2\xi - \frac{3\rho\epsilon^2\dot{\alpha}}{(\rho^2 + c^2\xi^2)}\right) + O_2 \tag{4.12}$$

$$\Psi_4 = \frac{\bar{\zeta}^2 \exp(2\underset{0}{u})}{(\rho + ic\xi)}\left(9\tilde{m}\dot{\alpha}^2 - 12\epsilon^2\dot{\alpha}^3\xi + \frac{6\epsilon^2\dot{\alpha}^2}{(\rho+ic\xi)}\right) + O_2$$

$$\Phi_1 = \frac{-\epsilon}{e(\rho - ic\xi)^2} + O_{3/2}$$

$$\Phi_2 = \frac{-2\epsilon\dot{\alpha}\bar{\zeta}\exp(\underset{0}{u})}{(\rho + ic\xi)} + O_{3/2}. \tag{4.13}$$

These are only singular on $\rho = 0$, $\Omega = 0$. The linearised Weyl tensor is Petrov type II. The first-order functions $R(\sigma)$ and $\tau(\sigma)$ remain undetermined. However, these do not appear in (4.12) and (4.13) and, as Hogan and Imaeda (1979a,b) show, they can be removed by a gauge transformation

$$\begin{aligned} \underset{1}{K} &\to \underset{1}{K} - R(\sigma) \\ \underset{1}{u} &\to \underset{1}{u} + \tfrac{1}{2}R(\sigma) - \tau(\sigma)\xi. \end{aligned} \tag{4.14}$$

We apply (4.14) to (4.9).

5. Discussion

Starting with a line element belonging to the Robinson *et al* (1969) family, which is interpreted as describing the exterior field of an axially symmetric charged source, rotating slowly about its symmetry axis and moving with arbitrary acceleration along its symmetry axis, we have shown that if the linearised field of the body and the linearised twist of the degenerate principal null direction of the Weyl tensor are free from 'wire' singularities then either the body performs runaway motion from an unaccelerated state in the infinite past or its acceleration vanishes and, to first order, its mass and charge are constant. Its rotation is either uniform or singular in the infinite past and uniform in the infinite future.

The final form of the linearised line element is given by (2.1a) with

$$\begin{aligned} d\Sigma &= -\frac{ic(1-\xi^2)}{2\zeta}\, d\zeta + \frac{ic(1-\xi^2)}{2\bar{\zeta}}\, d\bar{\zeta} - d\sigma + O_2 \\ P &= \tfrac{1}{2} e^{\alpha}(1-\xi)[1 - \tfrac{3}{2}\epsilon^2\dot{\alpha}^2(1-\xi^2)](\rho + ic\xi) + O_2 \\ Z &= -\frac{i(1-\xi^2)}{2\zeta}[c + \rho(\dot{c} - 2c\dot{\alpha}\xi)] + O_2 \\ S &= \rho[\dot{\alpha}\xi + \tfrac{9}{2}\tilde{m}\dot{\alpha}^2 + 3\epsilon^2\dot{\alpha}^3\xi(1-\xi^2)] - \tfrac{1}{2}(1 + 6\epsilon^2\dot{\alpha}^2\xi^2) \\ &\quad + [\rho(\tilde{m} + 2\epsilon^2\dot{\alpha}\xi) + \epsilon^2](\rho^2 + c^2\xi^2)^{-1} + O_2 \end{aligned} \tag{5.1}$$

where ξ, α and c are given by (4.3), (4.10) and (4.11) and $\tilde{m}$ and ϵ are constant. If $c'' \neq 0$ in (4.11), we note that $-\sigma_1 < \sigma < \sigma_1$ so that the singularities in c and $\dot{\alpha}$, at past and future infinity respectively, are not allowed to develop. When $c'' = 0$, $-\infty < \sigma < \sigma_0$, and as $\sigma \to -\infty$ we recover the Newman *et al* (1965) solution. If in addition we put $c' = 0$, we obtain the Reissner–Nordstrom solution for a static charge.

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